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### **RESEARCH ARTICLE**

## On Decreasing of Quantity of Radiation Defects in Working Area of an Integrated Circuit

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### ABSTRACT

In this paper, we introduce an approach to organize a drain of radiation defects, which were generated during ion doping and other types of radiation processing of the working area of integrated circuits, manufactured in the framework of a multilayer structure. The approach based on the difference of properties of materials of the layers of the considered multilayer structure. An analytical approach for analysis of mass and heat transfer in a multilayer structures was introduced with account the spatial and temporal variations of their parameters, as well as the nonlinearity of the processes under consideration.

**Key words:** Analytical approach for prognosis, Drain of radiation defects, Integrated circuits, Multilayer structures

### INTRODUCTION

At present, one of the intensively solving problems is increasing of performance of solid-state electronics devices (diodes, field-effect, and bipolar transistors...,).<sup>[1-6]</sup> Furthermore, the influence of various types of radiation processing on semiconductor materials is currently being investigated.<sup>[7-9]</sup> Different approaches to reduce the influence of the radiation processing on the considered materials are revealed.<sup>[10-12]</sup> In this paper, we consider a multilayer structure, which is presented on Figure 1. Next, the ion doping of the epitaxial layer is considered to manufacture several elements of integrated circuit. With time, annealing of radiation defects was considered. Main aim of this paper is to analyze the redistribution and interaction of point radiation defects, as well as their simplest complexes in the materials of the considered multilayer structure after radiation exposure.

### **METHODS OF SOLUTION**

To solve our aim, we calculate and analyzed the distribution of concentration of point radiation defects in space and time in the considered multilayer structure. We calculate the distribution by solving the second Fick's law in the following form Zhang and Bower, Landau and Lifshits, Kitayama *et al.*, Fahey *et al.*, Vinetskiy and Kholodar<sup>[13-17]</sup>.

$$\frac{\partial I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_{I,V}(x, y, z, T) \times I^2(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, Z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, Z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, Z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, Z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, Y, Z, t) + \frac{\partial}{\partial y} \int_0^{L_z} I(x, Y, W, t) dW \right]$$

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Figure 1: Three-layer structure, which includes into itself a substrate, epitaxial layers, and buffer layer (view from side)

$$+\Omega \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_{S} \mu (x, y, z, t) \int_{0}^{L_{t}} I(x, y, W, t) dW \right]$$

$$(1)$$

$$\frac{\partial V(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{V} (x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{V} (x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{V} (x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{V,V} (x, y, z, T) V^{2} (x, y, z, t) - k_{I,V} (x, y, z, T) \times \left[ I(x, y, z, t) V(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left[ \frac{D_{VS}}{kT} \nabla_{S} \mu (x, y, z, t) \int_{0}^{L_{t}} V(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{VS}}{kT} \nabla_{S} \mu (x, y, z, t) \int_{0}^{L_{t}} V(x, y, W, t) dW \right]$$

with boundary and initial conditions

$$\frac{\partial I(x, y, z, t)}{\partial x}\Big|_{x=0} = 0, \ \frac{\partial I(x, y, z, t)}{\partial x}\Big|_{x=L_x} = 0, \ \frac{\partial I(x, y, z, t)}{\partial y}\Big|_{y=0} = 0, I(x, y, z, 0) =$$

$$=f_1(x, y, z), \ \frac{\partial I(x, y, z, t)}{\partial y}\Big|_{y=L_y} = 0, \ \frac{\partial I(x, y, z, t)}{\partial z}\Big|_{z=0} = 0, \ \frac{\partial I(x, y, z, t)}{\partial z}\Big|_{z=L_z} = 0,$$

$$\frac{\partial V(x, y, z, t)}{\partial x}\Big|_{x=0} = 0, \ \frac{\partial V(x, y, z, t)}{\partial x}\Big|_{x=L_x} = 0, \ \frac{\partial V(x, y, z, t)}{\partial y}\Big|_{y=0} = 0, V(x, y, z, 0)$$

$$=f_V(x, y, z), \ \frac{\partial V(x, y, z, t)}{\partial y}\Big|_{y=L_y} = 0, \ \frac{\partial V(x, y, z, t)}{\partial z}\Big|_{z=0} = 0, \ \frac{\partial V(x, y, z, t)}{\partial z}\Big|_{z=0} = 0.$$
(2)

Here are I(x,y,z,t) is the spatiotemporal distribution of concentration of radiation interstitials with the equilibrium distribution  $I^*$ ; V(x,y,z,t) is the spatiotemporal distribution of concentration of radiation vacancies with the equilibrium distribution  $V^*$ ;  $D_I(x,y,z,T)$ ,  $D_V(x,y,z,T)$ ,  $D_{IS}(x,y,z,T)$ , and  $D_{VS}(x,y,z,T)$  are

the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms  $V^2(x,y,z,t)$  and P(x,y,z,t) correspond to generation of divacancies and diinterstitials, respectively (see, for example, <sup>[17]</sup> and appropriate references in this book);  $k_{I,V}(x,y,z,T)$ ,  $k_{I,J}(x,y,z,T)$ , and  $k_{V,V}(x,y,z,T)$  are the parameters of recombination of point radiation defects and generation of their complexes;  $\Omega$  is the atomic volume of dopant;  $\nabla_s$  is the symbol of surficial gradient;  $\int_{0}^{L_s} I(x,y,z,t) dz$  and  $\int_{0}^{L_s} V(x,y,z,t) dz$  are the surficial concentrations of interstitials and vacancies on interface between layers of heterostructure (in this situation, we assume that Z-axis is perpendicular to interface between layers of heterostructure);  $\mu(x,y,z,t)$  is the chemical potential due to the presence of mismatch-induced stress in the considered multilayer structure. Distributions of divacancies  $\Phi_V(x,y,z,t)$  and diinterstitials  $\Phi_I(x,y,z,t)$  in space and time could be calculated by solving the following system of equations<sup>[16,17]</sup>.

$$\frac{\partial}{\partial t} \frac{\Phi_{I}(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_{I}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{I}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_{I}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{I}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial x} \left[ D_{\Phi_{I}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{I}(x, y, z, t)}{\partial y} \right] + \Omega \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{I}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{I}(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{I}S}}{kT} \nabla_{S} \mu(x, y, z, T) I^{2}(x, y, z, t) \right]$$

$$+k_{I}(x,y,z,T)I(x,y,z,t)$$
(3)

$$\frac{\partial}{\partial t} \frac{\Phi_{V}(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_{V}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{V}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_{V}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{V}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial x} \left[ D_{\Phi_{V}}(x, y, z, T) \frac{\partial}{\partial t} \frac{\Phi_{V}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, T) V^{2}(x, y, z, t) \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \int_{0}^{L_{z}} \Phi_{V}(x, y, W, t) dW \right] + \frac{\partial}{\partial y} \left[ \frac$$

 $+k_{V}(x,y,z,T)V(x,y,z,t)$ 

with boundary and initial conditions

$$\frac{\partial \Phi_I(x, y, z, t)}{\partial x}\bigg|_{x=0} = 0, \ \frac{\partial \Phi_I(x, y, z, t)}{\partial x}\bigg|_{x=L_x} = 0, \ \frac{\partial \Phi_I(x, y, z, t)}{\partial y}\bigg|_{y=0} = 0,$$

$$\frac{\partial \Phi_{I}(x, y, z, t)}{\partial y}\bigg|_{y=L_{y}} = 0, \frac{\partial \Phi_{I}(x, y, z, t)}{\partial z}\bigg|_{z=0} = 0, \frac{\partial \Phi_{I}(x, y, z, t)}{\partial z}\bigg|_{z=L_{z}} = 0,$$

$$\frac{\partial \Phi_{V}(x, y, z, t)}{\partial x}\bigg|_{x=0} = 0, \frac{\partial \Phi_{V}(x, y, z, t)}{\partial x}\bigg|_{x=L_{x}} = 0, \frac{\partial \Phi_{V}(x, y, z, t)}{\partial y}\bigg|_{y=0} = 0,$$
(4)

$$\frac{\partial \Phi_{V}(x, y, z, t)}{\partial y} \bigg|_{y=L_{y}} = 0, \frac{\partial \Phi_{V}(x, y, z, t)}{\partial z} \bigg|_{z=0} = 0, \frac{\partial \Phi_{V}(x, y, z, t)}{\partial z} \bigg|_{z=L_{z}} = 0,$$

 $\Phi_{I}(x,y,z,0) = f \Phi_{I}(x,y,z), \ \Phi_{V}(x,y,z,0) = f \Phi_{V}(x,y,z).$ 

Here  $D_{\phi l}(x,y,z,T)$ ,  $D_{\phi l}(x,y,z,T)$ ,  $D_{\phi lS}(x,y,z,T)$ , and  $D_{\phi VS}(x,y,z,T)$  are the coefficients of volumetric and surficial diffusions of complexes of radiation defects;  $k_l(x,y,z,T)$  and  $k_l(x,y,z,T)$  are the parameters of decay of complexes of radiation defects. Chemical potential  $\mu_1$  in Eq.(1) could be determined by the following relation <sup>[13]</sup>.

$$\mu_1 = E(z) \Omega \sigma_{ij} \left[ u_{ij}(x, y, z, t) + u_{ji}(x, y, z, t) \right] / 2, \tag{5}$$

where E(z) is the Young modulus,  $\sigma_{ij}$  is the stress tensor;  $u_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$  is the deformation tensor;

 $u_i, u_j$  are the components  $u_x(x,y,z,t), u_y(x,y,z,t)$ , and  $u_z(x,y,z,t)$  of the displacement vector  $\vec{u}(x, y, z, t); x_i, x_j$  are the coordinate x, y, and z. The Eq. (5) could be transformed to the following form

$$\mu (x, y, z, t) = \left[ \frac{\partial u_i(x, y, z, t)}{\partial x_j} + \frac{\partial u_j(x, y, z, t)}{\partial x_i} \right] \left\{ \frac{1}{2} \left[ \frac{\partial u_i(x, y, z, t)}{\partial x_j} + \frac{\partial u_j(x, y, z, t)}{\partial x_i} \right] - \varepsilon_0 \delta_{ij} + \frac{\sigma (z) \delta_{ij}}{1 - 2\sigma (z)} \left[ \frac{\partial u_k(x, y, z, t)}{\partial x_k} - 3\varepsilon_0 \right] - K(z) \beta (z) \left[ T(x, y, z, t) - T_0 \right] \delta_{ij} \right\} \frac{\Omega}{2} E(z),$$

where  $\sigma$  is Poisson coefficient;  $\varepsilon_0 = (a_s - a_{EL})/a_{EL}$  is the mismatch parameter;  $a_s, a_{EL}$  are lattice distances of the substrate and the epitaxial layer; *K* is the modulus of uniform compression;  $\beta$  is the coefficient of thermal expansion;  $T_r$  is the equilibrium temperature, which coincides (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equation<sup>[14]</sup>

$$\rho(z)\frac{\partial^{2} u_{x}(x, y, z, t)}{\partial t^{2}} = \frac{\partial \sigma_{xx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{xy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{xz}(x, y, z, t)}{\partial z}$$

$$\rho(z)\frac{\partial^{2} u_{y}(x, y, z, t)}{\partial t^{2}} = \frac{\partial \sigma_{yx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{yy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{yz}(x, y, z, t)}{\partial z}$$

$$\rho(z)\frac{\partial^{2} u_{z}(x, y, z, t)}{\partial z^{2}} = \frac{\partial \sigma_{zx}(x, y, z, t)}{\partial x} + \frac{\partial \sigma_{zy}(x, y, z, t)}{\partial y} + \frac{\partial \sigma_{zz}(x, y, z, t)}{\partial z}$$

$$\rho(z)\frac{\partial u_z(x,y,z,t)}{\partial t^2} = \frac{\partial \partial u_z(x,y,z,t)}{\partial x} + \frac{\partial \partial u_z(x,y,z,t)}{\partial y} + \frac{\partial \partial u_z(x,y,z,t)}{\partial z},$$

where 
$$\sigma_{ij} = \frac{E(z)}{2[1+\sigma(z)]} \left[ \frac{\partial u_i(x,y,z,t)}{\partial x_j} + \frac{\partial u_j(x,y,z,t)}{\partial x_i} - \frac{\delta_{ij}}{3} \frac{\partial u_k(x,y,z,t)}{\partial x_k} \right] + K(z) \delta_{ij} \times C_{ij}$$

 $\times \frac{\partial u_k(x, y, z, t)}{\partial x_k} - \beta(z) K(z) [T(x, y, z, t) - T_r], \rho(z) \text{ is the density of materials of heterostructure, } \delta_{ij} \text{ Is}$ 

the Kronecker symbol. With account the relation for  $\sigma_{ij}$  last system of equation could be written as

$$\rho\left(z\right)\frac{\partial^{2} u_{x}\left(x,y,z,t\right)}{\partial t^{2}} = \left\{K\left(z\right) + \frac{5E\left(z\right)}{6\left[1+\sigma\left(z\right)\right]}\right\}\frac{\partial^{2} u_{x}\left(x,y,z,t\right)}{\partial x^{2}} + \left\{K\left(z\right) - \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right\} \times \frac{\partial^{2} u_{y}\left(x,y,z,t\right)}{\partial x \partial y} + \frac{E\left(z\right)}{2\left[1+\sigma\left(z\right)\right]}\left[\frac{\partial^{2} u_{y}\left(x,y,z,t\right)}{\partial y^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right] \times \frac{\partial^{2} u_{y}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right] \times \frac{\partial^{2} u_{y}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right] \times \frac{\partial^{2} u_{y}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right] \times \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{E\left(z\right)}{3\left[1+\sigma\left(z\right)\right]}\right] \times \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \left[K\left(z\right) + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}}\right] + \frac{\partial^{2} u_{z}\left(x,y,z,t\right)}{\partial z^{2}} + \frac{\partial^{$$

$$\times \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial x \partial z} - K(z)\beta(z)\frac{\partial T(x, y, z, t)}{\partial x}$$

$$\rho(z)\frac{\partial^{2} u_{y}(x, y, z, t)}{\partial t^{2}} = \frac{E(z)}{2[1+\sigma(z)]} \left[ \frac{\partial^{2} u_{y}(x, y, z, t)}{\partial x^{2}} + \frac{\partial^{2} u_{x}(x, y, z, t)}{\partial x \partial y} \right] - \frac{\partial T(x, y, z, t)}{\partial y} \times$$

$$\times K(z)\beta(z) + \frac{\partial}{\partial z} \left\{ \frac{E(z)}{2[1+\sigma(z)]} \left[ \frac{\partial u_{y}(x, y, z, t)}{\partial z} + \frac{\partial u_{z}(x, y, z, t)}{\partial y} \right] \right\} + \frac{\partial^{2} u_{y}(x, y, z, t)}{\partial y^{2}} \times$$

$$\times \left\{ \frac{5E(z)}{12[1+\sigma(z)]} + K(z) \right\} + \left\{ K(z) - \frac{E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^{2} u_{y}(x, y, z, t)}{\partial y \partial z} + K(z) \frac{\partial^{2} u_{y}(x, y, z, t)}{\partial x \partial y} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial x \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial y \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial x \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial y \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial x \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial y \partial z} + \frac{\partial^{2} u_{z}(x, y, z, t)}{\partial z \partial z} + \frac{\partial^{$$

Conditions for the system of Eq. (6) could be written in the form

$$\frac{\partial \vec{u}(0, y, z, t)}{\partial x} = 0; \quad \frac{\partial \vec{u}(L_x, y, z, t)}{\partial x} = 0; \quad \frac{\partial \vec{u}(x, 0, z, t)}{\partial y} = 0; \quad \frac{\partial \vec{u}(x, L_y, z, t)}{\partial y} = 0;$$

$$\frac{\partial \vec{u}(x, y, 0, t)}{\partial z} = 0; \quad \frac{\partial \vec{u}(x, y, L_z, t)}{\partial z} = 0; \quad \vec{u}(x, y, z, 0) = \vec{u}_0; \quad \vec{u}(x, y, z, \infty) = \vec{u}_0$$

We calculate distributions of concentrations of radiation defects in space and time by solving the Eqs.(1) and (3) framework standard algorithm of method of averaging of function corrections<sup>[18-20]</sup>. Previously, we transform the Eqs. (1) and (3) to the following form with account initial distributions of the considered concentrations

$$\frac{\partial I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \Omega \frac{\partial}{\partial x} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \Omega \frac{\partial}{\partial y} \left[ \frac{D_{IS}}{kT} \nabla_S \mu(x, y, z, t) \int_0^{L_z} I(x, y, Z, t) \int_0^{L_z} I(x, y, W, t) dW \right] - k_{I,I}(x, y, z, T) I^2(x, y, z, t) -$$

Furthermore, we replace concentrations of radiation defects in the right sides of on their not yet known average values 
$$\alpha_{12}$$
. In this situation, we obtain equation

Eqs. (1a) and (3a)s for the first-order 0 approximations of the required concentrations in the following form  $\overline{2}$ 

$$\frac{\partial I_{I}(x, y, z, t)}{\partial t} = \alpha_{II} z \Omega \frac{\partial}{\partial x} \left[ \frac{D_{IS}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + \alpha_{II} \Omega \frac{\partial}{\partial y} \left[ z \frac{D_{IS}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + f_{I}(x, y, z) \delta(t) - \alpha_{II}^{2} k_{I,I}(x, y, z, T) - \alpha_{II} \alpha_{IV} k_{I,V}(x, y, z, T)$$

$$(1b)$$

$$\frac{\partial V_{1}(x, y, z, t)}{\partial t} = \alpha_{1V} z \Omega \frac{\partial}{\partial x} \left[ \frac{D_{VS}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + \alpha_{1V} \Omega \frac{\partial}{\partial y} \left[ z \frac{D_{VS}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + f_{V}(x, y, z) \delta(t) - \alpha_{1V}^{2} k_{V,V}(x, y, z, T) - \alpha_{1I} \alpha_{1V} k_{I,V}(x, y, z, T)$$

$$\frac{\partial \Phi_{1I}(x, y, z, t)}{\partial t} = \alpha_{1\Phi_{I}} z \Omega \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{I}S}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + \alpha_{1\Phi_{I}} z \Omega \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{I}S}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + f_{\Phi_{I}}(x, y, z) \delta(t) + k_{I}(x, y, z, T) I(x, y, z, t) + k_{I,I}(x, y, z, T) I^{2}(x, y, z, t)$$
(3b)

$$\frac{\partial \Phi_{1V}(x, y, z, t)}{\partial t} = \alpha_{1\Phi_{V}} z \Omega \frac{\partial}{\partial x} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + \alpha_{1\Phi_{V}} z \Omega \frac{\partial}{\partial y} \left[ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu(x, y, z, t) \right] + f_{\Phi_{V}}(x, y, z) \delta(t) + k_{V}(x, y, z, T) V(x, y, z, t) + k_{V,V}(x, y, z, T) V^{2}(x, y, z, t).$$

Integration of the left and right sides of the Eqs. (1b) and (3b) on time gives us a possibility to obtain relations for above approximation in the final form

$$I_{1}(x,y,z,t) = \alpha_{1I}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + \alpha_{1I}z \Omega \frac{\partial}{\partial y} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + f_{I}(x,y,z) - \alpha_{1I}^{2} \int_{0}^{t} k_{I,I}(x,y,z,T) d\tau - \alpha_{1I}\alpha_{1V} \int_{0}^{t} k_{I,V}(x,y,z,T) d\tau$$

$$(3c)$$

$$V_{1}(x,y,z,t) = \alpha_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + \alpha_{1V}z \Omega \frac{\partial}{\partial y} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + f_{V}(x,y,z,\tau) d\tau + f_{V}(x,y,z,\tau) d\tau + \alpha_{1V}z \Omega \frac{\partial}{\partial y} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + f_{V}(x,y,z,\tau) d\tau + f_{V}(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS}}{kT} \nabla_{S} \mu(x,y,z,\tau) d\tau + g_{1V}z \Omega \frac{\partial}{\partial x} \int_{0}^{t} \frac{D_{IS$$

$$\times \alpha_{1\Phi_{V}}z + f_{\Phi_{V}}(x, y, z) + \int_{0}^{0} k_{V}(x, y, z, T) V(x, y, z, \tau) d\tau + \int_{0}^{0} k_{V,V}(x, y, z, T) V^{2}(x, y, z, \tau) d\tau$$

We determine average values of the first-order approximations of concentrations of radiation defects by the following standard relation<sup>[18-20]</sup>

$$\alpha_{1\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \rho_1(x, y, z, t) dz dy dx dt$$
(7)

Substitution of the relations (1c) and (3c) into relation (7) gives us a possibility to obtain required average values in the following form

$$\alpha_{1C} = \frac{1}{L_x L_y L_z} \int_{0}^{L_x} \int_{0}^{L_y} \int_{0}^{L_z} f_C(x, y, z) dz dy dx, \quad \alpha_{1I} = \sqrt{\frac{(a_3 + A)^2}{4a_4^2}} - 4\left(B + \frac{\Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4}\right) - -\frac{a_3 + A}{4a_4}$$

$$\alpha_{1V} = \frac{1}{S_{IV00}} \left[\frac{\Theta}{\alpha_{1I}} \int_{0}^{L_x} \int_{0}^{L_y} \int_{0}^{L_z} f_I(x, y, z) dz dy dx - \alpha_{1I} S_{II00} - \Theta L_x L_y L_z\right],$$

where 
$$S_{\rho\rho\,ij} = \int_{0}^{\infty} (\Theta - t) \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} k_{\rho,\rho} (x, y, z, T) I_{1}^{i}(x, y, z, t) V_{1}^{j}(x, y, z, t) dz dy dx dt$$
,  $a_{4} = S_{II00} \times C_{1}^{i}(x, y, z, t) V_{1}^{j}(x, y, z, t) dz dy dx dt$ ,  $a_{4} = S_{II00} \times C_{1}^{i}(x, y, z, t) V_{1}^{j}(x, y, z, t) dz dy dx dt$ 

$$\times \left(S_{IV00}^2 - S_{II00}S_{VV00}\right), \ a_3 = S_{IV00}S_{II00} + S_{IV00}^2 - S_{II00}S_{VV00}, \ a_2 = \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_V(x, y, z) dz \, dy \, dx \times \int_0^{L_y} f_V(x, y, z) dz \, dy \, dx = \int_0^{L_y} \int_0^{L_y} \int_0^{L_y} f_V(x, y, z) dz \, dy \, dx = \int_0^{L_y} \int_0^{L_y} \int_0^{L_y} \int_0^{L_y} f_V(x, y, z) dz \, dy \, dx = \int_0^{L_y} \int_0^{L_y}$$

$$\times S_{IV00}S_{IV00}^{2} + S_{IV00}\Theta L_{x}^{2}L_{y}^{2}L_{z}^{2} + 2S_{VV00}S_{II00}\int_{0}^{L_{x}}\int_{0}^{L_{y}}\int_{0}^{L_{z}}f_{I}(x, y, z)dzdydx - \Theta L_{x}^{2}L_{y}^{2}L_{z}^{2}S_{VV00} - \frac{1}{2}\int_{0}^{L_{x}}f_{I}(x, y, z)dzdydx - \Theta L_{x}^{2}L_{y}^{2}L_{z}^{2}S_{VV00} - \frac{1}{2}\int_{0}^{L_{x}}f_{I}(x, y, z)dzdydx - \frac{1}{2}\int_{0}^{$$

$$-S_{IV00}^{2} \int_{0}^{L_{x}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} \int_{0}^{L_{y}} f_{I}(x, y, z) dz dy dx, a_{1} = S_{IV00} \int_{0}^{L_{x}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{I}(x, y, z) dz dy dx, a_{0} = S_{VV00} \times S_{VV00} = S_{VV00} + S_{VV00}$$

$$\times \left[ \int_{0}^{L_{x}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} f_{I}(x, y, z) dz dy dx \right]^{2}, A = \sqrt{8y + \Theta^{2} \frac{a_{3}^{2}}{a_{4}^{2}} - 4\Theta \frac{a_{2}}{a_{4}}}, B = \frac{\Theta a_{2}}{6a_{4}} + \sqrt[3]{\sqrt{q^{2} + p^{3}} - q} - \sqrt[3]{\sqrt{q^{2} + p^{3}} + q}, q = \frac{\Theta^{3}a_{2}}{24a_{4}^{2}} \left( 4a_{0} - \Theta L_{x}L_{y}L_{z}\frac{a_{1}a_{3}}{a_{4}} \right) - \Theta^{2}\frac{a_{0}}{8a_{4}^{2}} \left( 4\Theta a_{2} - \Theta^{2}\frac{a_{3}^{2}}{a_{4}} \right) - \frac{\Theta^{3}a_{2}^{3}}{54a_{4}^{3}} - L_{x}^{2}L_{y}^{2}L_{z}^{2}\frac{\Theta^{4}a_{1}^{2}}{8a_{4}^{2}}, p = \Theta^{2}\frac{4a_{0}a_{4} - \Theta L_{x}L_{y}L_{z}a_{1}a_{3}}{12a_{4}^{2}} - \frac{\Theta a_{2}}{18a_{4}},$$

$$\alpha_{1\Phi_{I}} = \frac{R_{I1}}{\Theta L_{x}L_{y}L_{z}} + \frac{S_{II20}}{\Theta L_{x}L_{y}L_{z}} + \frac{1}{L_{x}L_{y}L_{z}} \int_{0}^{L_{x}} \int_{0}^{L_{y}} \int_{0}^{L_{z}} \int_{0}^{L_{z}} f_{\Phi_{I}}(x, y, z) dz dy dx$$

$$\alpha_{1\Phi_{\nu}} = \frac{R_{\nu_1}}{\Theta L_x L_y L_z} + \frac{S_{\nu\nu_2 0}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z} \int_0^{L_z} \int_0^{L_z} \int_0^{L_z} f_{\Phi_{\nu}} \left( x, y, z \right) dz dy dx ,$$

where 
$$R_{\rho i} = \int_{0}^{\Theta} (\Theta - t) \int_{0}^{L_x L_y} \int_{0}^{L_z} \int_{0}^{L_z} k_I(x, y, z, T) I_1^i(x, y, z, t) dz dy dx dt$$
.

We determine approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of the method of averaging of function corrections <sup>[18-20]</sup>. In the framework of the procedure to determine approximations of the n-th order of concentrations of radiation defects we replace the required concentrations in the Eqs. (*1c*), (*3c*) on the following sum  $\alpha_{np} + \rho_{n-1}(x,y,z,t)$ . The replacement leads to the following transformation of the appropriate equations.

$$\begin{split} \frac{\partial}{\partial t_{z}} \left( (x,y,z,t) - \frac{\partial}{\partial t_{z}} \left[ D_{l} (x,y,z,T) \frac{\partial}{\partial t_{z}} \frac{I_{l}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{l} (x,y,z,T) \frac{\partial}{\partial t_{z}} \frac{I_{l}(x,y,z,t)}{\partial y} \right] + \\ + \frac{\partial}{\partial z} \left[ D_{l} (x,y,z,T) \frac{\partial}{\partial t_{z}} \frac{I_{l}(x,y,z,t)}{\partial z} \right] - k_{l,l} (x,y,z,T) \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right]^{2} - k_{l,l} (x,y,z,T) \times \\ \times \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right] \left[ \alpha_{l,l} + V_{l} (x,y,z,t) \right] + \Omega \frac{\partial}{\partial x} \left\{ \nabla_{S} \mu (x,y,z,t) \right]_{0}^{l} \left[ \alpha_{2,l} + I_{l} (x,y,w,t) \right] dW \times \\ \times \frac{D_{R}}{kT} \right\} + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{R}}{kT} \nabla_{S} \mu (x,y,z,t) \right]_{0}^{l} \left[ \left( \alpha_{2,l} + I_{l} (x,y,w,t) \right] \right] dW \right\} \tag{1d}$$

$$\frac{\partial}{\partial t} \frac{V_{l}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{l} (x,y,z,T) \frac{\partial V_{l}(x,y,z,t)}{\partial z} \right] + \frac{\partial}{\partial y} \left[ D_{l} (x,y,z,T) \frac{\partial V_{l}(x,y,z,t)}{\partial y} \right] + \\ + \frac{\partial}{\partial z} \left[ D_{r} (x,y,z,T) \frac{\partial V_{l}(x,y,z,t)}{\partial z} \right] - k_{r,r} (x,y,z,T) \left[ \alpha_{1r} + V_{l} (x,y,z,t) \right]^{2} - k_{l,r} (x,y,z,T) \times \\ \times \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right] \left[ \alpha_{l,r} + V_{l} (x,y,z,t) \right] - k_{r,r} (x,y,z,T) \left[ \alpha_{1r} + V_{l} (x,y,z,t) \right]^{2} - k_{l,r} (x,y,z,T) \times \\ \times \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right] \left[ \alpha_{l,r} + V_{l} (x,y,z,t) \right] + \Omega \frac{\partial}{\partial x} \left\{ \nabla_{S} \mu (x,y,z,t) \right]^{2} - k_{l,r} (x,y,z,T) \times \\ \times \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right] \left[ \alpha_{l,r} + V_{l} (x,y,z,t) \right] + \Omega \frac{\partial}{\partial x} \left\{ \nabla_{S} \mu (x,y,z,t) \right]^{2} - k_{l,r} (x,y,z,T) \times \\ \times \left[ \alpha_{l,l} + I_{l} (x,y,z,t) \right] \left[ \alpha_{l,r} + V_{l} (x,y,z,t) \right]^{2} \left[ \alpha_{2,r} + V_{l} (x,y,w,t) \right] dW \times \\ \times \frac{D_{R}}{kT} + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{R}}{kT} \nabla_{S} \mu (x,y,z,t) \right]^{2} \left[ \alpha_{2,q} + \Phi_{l,l} (x,y,w,t) \right] dW \right\} + k_{l,l} (x,y,z,T) \frac{\partial}{\partial u_{l}} (x,y,z,t) + \\ + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{q,s}}{kT} \nabla_{S} \mu (x,y,z,t) \right]^{l}_{0} \left[ \alpha_{2,q} + \Phi_{l,l} (x,y,w,t) \right] dW \right\} + k_{l,l} (x,y,z,T) \frac{\partial}{\partial u_{l}} (x,y,z,t) + \\ + \Omega \frac{\partial}{\partial x} \left[ D_{q_{r}} (x,y,z,t) \frac{\partial}{\partial u_{l}} \left[ \alpha_{2,q_{r}} + \Phi_{l,l} (x,y,w,t) \right] dW \right\} + k_{l} \frac{\partial}{\partial y} \left[ D_{q_{r}} (x,y,z,t) + \\ + \Omega \frac{\partial}{\partial x} \left[ D_{q_{r}} (x,y,z,t) \frac{\partial}{\partial u_{r}} \left[ \alpha_{2,q_{r}} + \Phi_{l,l} (x,y,w,t) \right] dW \right] + k_{r,r} (x,y,z,T) \frac{\partial}{\partial u_{r}} (x,y,z,t) + \\ + \Omega \frac{\partial}{\partial x} \left[ \frac{D_{q,s}}{kT} \nabla$$

$$+\Omega \frac{\partial}{\partial y} \left\{ \frac{D_{\Phi_{V}S}}{kT} \nabla_{S} \mu \left( x, y, z, t \right) \int_{0}^{L_{z}} \left[ \alpha_{2\Phi_{V}} + \Phi_{1V} \left( x, y, W, t \right) \right] dW \right\} + k_{V} \left( x, y, z, T \right) V \left( x, y, z, t \right) + \frac{\partial}{\partial z} \left[ D_{\Phi_{V}} \left( x, y, z, T \right) \frac{\partial \Phi_{1V} \left( x, y, z, t \right)}{\partial z} \right] + f_{\Phi_{V}} \left( x, y, z \right) \delta \left( t \right).$$

Integration of the left and the right sides of Eqs. (1d) and (3d) gives us a possibility to obtain relations for the required concentrations in the final form

$$\begin{split} I_{2}(x,y,z,t) &= \frac{\partial}{\partial x} \int_{0}^{t} D_{l}(x,y,z,T) \frac{\partial I_{1}(x,y,z,\tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} D_{l}(x,y,z,T) \frac{\partial I_{1}(x,y,z,\tau)}{\partial y} d\tau + \\ &+ \frac{\partial}{\partial z} \int_{0}^{t} D_{l}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,z,\tau) \right] \left[ \alpha_{2y}, + V_{1}(x,y,z,T) \right] \left[ \alpha_{2t}, + I_{1}(x,y,z,\tau) \right]^{2} d\tau - \\ &- \int_{0}^{t} k_{t,t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,z,\tau) \right] \left[ \alpha_{2y}, + V_{1}(x,y,z,\tau) \right] d\tau + \frac{\partial}{\partial x} \int_{0}^{t} \nabla_{s} \mu (x,y,z,\tau) \times \\ &\times \Omega \frac{D_{S}}{kT} \int_{0}^{t} \left[ \alpha_{2t}, + I_{1}(x,y,W,\tau) \right] dW d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \nabla_{s} \mu (x,y,z,\tau) \int_{0}^{t} \left[ \alpha_{2t}, + I_{1}(x,y,W,\tau) \right] \times \\ &\times \Omega \frac{D_{S}}{kT} dW d\tau + f_{t}(x,y,z) \end{split}$$
(1e)  
$$V_{2}(x,y,z,t) &= \frac{\partial}{\partial x} \int_{0}^{t} D_{t}(x,y,z,T) \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau - \int_{0}^{t} k_{t,t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,W,\tau) \right]^{2} d\tau - \\ &- \int_{0}^{t} k_{t,t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,Z,T) \right] \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} D_{t}(x,y,z,T) \frac{\partial V_{1}(x,y,z,\tau)}{\partial y} d\tau + \\ &+ \frac{\partial}{\partial z} \int_{0}^{t} D_{t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,Z,T) \right] \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau - \int_{0}^{t} k_{t,t}(x,y,z,T) \left[ \alpha_{2t}, + V_{1}(x,y,Z,T) \right] \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau + \\ &- \frac{\partial}{\partial z} \int_{0}^{t} D_{t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,Z,T) \right] \left[ \alpha_{2t} + V_{1}(x,y,Z,T) \right] \frac{\partial V_{1}(x,y,z,\tau)}{\partial z} d\tau - \\ &- \int_{0}^{t} k_{t,t}(x,y,z,T) \left[ \alpha_{2t}, + I_{1}(x,y,Z,T) \right] \left[ \alpha_{2t} + V_{1}(x,y,Z,T) \right] d\tau + \frac{\partial}{\partial x} \int_{0}^{t} \nabla_{s} \mu (x,y,z,\tau) \times \\ &\times \Omega \frac{D_{t,t}}{kT} \int_{0}^{t} \left[ \alpha_{2t}, + V_{1}(x,y,W,\tau) \right] dW d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \nabla_{s} \mu (x,y,z,\tau) \int_{0}^{t} \left[ \alpha_{2t}, + V_{1}(x,y,W,\tau) \right] \times \\ &\times \Omega \frac{D_{t,t}}{kT} dW d\tau + f_{t}(x,y,z) \\ &\Phi_{2t}(x,y,z,t) = \frac{\partial}{\partial x} \int_{0}^{t} D_{\phi_{t}}(x,y,z,T) \frac{\partial \Phi_{t,t}(x,y,z,\tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \frac{\partial \Phi_{t,t}(x,y,Z,\tau)}{\partial y} \times \\ &\times D_{\phi_{t}}(x,y,z,T) d\tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\phi_{t}}(x,y,z,T) \frac{\partial \Phi_{t,t}(x,y,z,\tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \frac{\partial \Phi_{t,t}(x,y,Z,\tau)}{\partial y} \times \\ &\times D_{\phi_{t}}(x,y,z,T) d\tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\phi_{t}}(x,y,z,T) \frac{\partial \Phi_{t,t}(x,y,z,\tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \frac{\partial \Phi_{t,t}(x,y,Z,\tau)}{\partial y} \times \\ \end{aligned}$$

$$\times \frac{D_{\Phi_{I}S}}{kT} \int_{0}^{L_{z}} \left[ \alpha_{2\Phi_{I}} + \Phi_{1I}\left(x, y, W, \tau\right) \right] dW d\tau + \Omega \frac{\partial}{\partial y} \int_{0}^{t} \frac{D_{\Phi_{I}S}}{kT} \int_{0}^{L_{z}} \left[ \alpha_{2\Phi_{I}} + \Phi_{1I}\left(x, y, W, \tau\right) \right] dW \times$$

$$\times \nabla_{S} \mu \left(x, y, z, \tau\right) d\tau + \int_{0}^{t} k_{I,I}\left(x, y, z, T\right) I^{2}\left(x, y, z, \tau\right) d\tau + \int_{0}^{t} k_{I}\left(x, y, z, \tau\right) I\left(x, y, z, \tau\right) d\tau +$$

$$+ f_{\Phi_{I}}\left(x, y, z\right)$$

$$(3e)$$

$$\Phi_{2\nu}(x,y,z,t) = \frac{\partial}{\partial x} \int_{0}^{t} D_{\Phi_{\nu}}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,\tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_{0}^{t} \frac{\partial \Phi_{1\nu}(x,y,z,\tau)}{\partial y} \times \\ \times D_{\Phi_{\nu}}(x,y,z,T) d\tau + \frac{\partial}{\partial z} \int_{0}^{t} D_{\Phi_{\nu}}(x,y,z,T) \frac{\partial \Phi_{1\nu}(x,y,z,\tau)}{\partial z} d\tau + \Omega \frac{\partial}{\partial x} \int_{0}^{t} \nabla_{s} \mu(x,y,z,\tau) \times \\ \times \frac{D_{\Phi_{\nu}s}}{kT} \int_{0}^{L_{z}} \left[ \alpha_{2\Phi_{\nu}} + \Phi_{1\nu}(x,y,W,\tau) \right] dW d\tau + \Omega \frac{\partial}{\partial y} \int_{0}^{t} \frac{D_{\Phi_{\nu}s}}{kT} \int_{0}^{L_{z}} \left[ \alpha_{2\Phi_{\nu}} + \Phi_{1\nu}(x,y,W,\tau) \right] dW \times \\ \times \nabla_{s} \mu(x,y,z,\tau) d\tau + \int_{0}^{t} k_{\nu,\nu}(x,y,z,T) V^{2}(x,y,z,\tau) d\tau + \int_{0}^{t} k_{\nu}(x,y,z,T) V(x,y,z,\tau) d\tau + \\ + f_{\Phi_{\nu}}(x,y,z)$$

Average values of the second-order approximations of required approximations using the following standard relation<sup>[18-20]</sup>

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[ \rho_2(x, y, z, t) - \rho_1(x, y, z, t) \right] dz dy dx dt$$
(8)

Substitution of the relations (1*e*), (3*e*) into relation (8) gives us a possibility to obtain relations for required average values  $\alpha_{2\rho}$ 

$$\begin{aligned} \alpha_{2C} &= 0, \, \alpha_{2\Phi I} = 0, \, \alpha_{2\Phi V} = 0, \, \alpha_{2V} = \sqrt{\frac{\left(b_3 + E\right)^2}{4b_4^2}} - 4\left(F + \frac{\Theta a_3 F + \Theta^2 L_x L_y L_z b_1}{b_4}\right) - \frac{b_3 + E}{4b_4}, \\ \alpha_{2I} &= \frac{C_V - \alpha_{2V}^2 S_{VV00} - \alpha_{2V} \left(2 S_{VV01} + S_{IV10} + \Theta L_x L_y L_z\right) - S_{VV02} - S_{IV11}}{S_{IV01} + \alpha_{2V} S_{IV00}}, \end{aligned}$$

where 
$$b_4 = \frac{1}{\Theta L_x L_y L_z} S_{IV00}^2 S_{VV00} - \frac{1}{\Theta L_x L_y L_z} S_{VV00}^2 S_{II00}, \ b_3 = -\frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + S_{IV10} + S_{IV10}) + S_{IV10} + S_{IV10}$$

$$+\Theta L_{x}L_{y}L_{z}\Big)+\frac{S_{IV00}S_{VV00}}{\Theta L_{x}L_{y}L_{z}}\Big(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_{x}L_{y}L_{z}\Big)+\frac{S_{IV00}^{2}}{\Theta L_{x}L_{y}L_{z}}\Big(2S_{VV01}+S_{IV10}$$

$$+\Theta L_{x}L_{y}L_{z}\Big)-\frac{S_{IV00}^{2}S_{IV10}}{\Theta^{3}L_{x}^{3}L_{y}^{3}L_{z}^{3}}, b_{2}=\frac{S_{II00}S_{VV00}}{\Theta L_{x}L_{y}L_{z}}(S_{VV02}+S_{IV11}+C_{V})-(S_{IV10}-2S_{VV01}+\Theta L_{x}L_{y}\times C_{V})$$

$$\times L_{z}^{2} + \frac{S_{IV01}S_{VV00}}{\Theta L_{x}L_{y}L_{z}} \left(\Theta L_{x}L_{y}L_{z} + 2S_{II10} + S_{IV01}\right) + \frac{S_{IV00}}{\Theta L_{x}L_{y}L_{z}} \left(S_{IV01} + 2S_{II10} + 2S_{IV01} + \Theta L_{x}L_{y} \times \frac{S_{IV01}S_{VV00}}{\Theta L_{x}L_{y}L_{z}}\right)$$

$$\begin{split} & \times L_{z} \Big) \Big( 2S_{VV01} + \Theta L_{x}L_{y}L_{z} + S_{IV10} \Big) - \frac{S_{IV00}^{2}}{\Theta L_{x}L_{y}L_{z}} \Big( C_{V} - S_{VV02} - S_{IV11} \Big) + \frac{C_{I}S_{IV00}^{2}}{\Theta^{2}L_{x}^{2}L_{y}^{2}L_{z}^{2}} - \frac{2S_{IV10}}{\Theta L_{x}L_{y}L_{z}} \times \\ & \times S_{IV00}S_{IV01}, \ b_{1} = S_{II00} \frac{S_{IV11} + S_{VV02} + C_{V}}{\Theta L_{x}L_{y}L_{z}} \Big( 2S_{VV01} + S_{IV10} + \Theta L_{x}L_{y}L_{z} \Big) + \frac{S_{IIV0}}{\Theta L_{x}L_{y}L_{z}} \Big( \Theta L_{x}L_{y} \times \\ & \times L_{z} + 2S_{II10} + S_{IIV01} \Big) \Big( 2S_{VV01} + S_{IV10} + \Theta LL_{y}L_{z} \Big) - \frac{S_{IV10}S_{IV01}^{2}}{\Theta L_{x}L_{y}L_{z}} - \frac{S_{IIV00}}{\Theta L_{x}L_{y}L_{z}} \Big( 3S_{IIV01} + 2S_{II10} + \\ & + \Theta L_{x}L_{y}L_{z} \Big) \Big( C_{V} - S_{VV02} - S_{IIV11} \Big) + 2C_{I}S_{IIV00}S_{IIV01}, \ b_{0} = \frac{S_{II00}}{\Theta L_{x}L_{y}L_{z}} \Big( S_{IIV00} + S_{VV02} \Big)^{2} - \frac{S_{IIV0}}{L_{x}L_{y}L_{z}} \times \\ & \times \frac{1}{\Theta} \Big( \Theta L_{x}L_{y}L_{z} + 2S_{II10} + S_{IIV01} \Big) \Big( C_{V} - S_{VV02} - S_{IIV11} \Big) + 2C_{I}S_{IIV01}^{2} - S_{IIV01} - S_{IIV01} \frac{C_{V} - S_{VV02} - S_{IIV11}}{\Theta L_{x}L_{y}L_{z}} \times \\ & \times \frac{1}{\Theta} \Big( \Theta L_{x}L_{y}L_{z} + 2S_{II10} + S_{IIV01} \Big) \Big( C_{V} - S_{VV02} - S_{IIV11} \Big) + 2C_{I}S_{IIV01}^{2} - S_{IIV01} \frac{C_{V} - S_{VV02} - S_{IIV11}}{\Theta L_{x}L_{y}L_{z}} \times \\ & \times \frac{1}{\Theta} \Big( \Theta L_{x}L_{y}L_{z} + 2S_{II10} + S_{IIV01} \Big) \Big( C_{V} - S_{VV02} - S_{IIV11} \Big) + 2C_{I}S_{IIV01}^{2} - S_{IIV01} \frac{C_{V} - S_{VV02} - S_{IIV11}}{\Theta L_{x}L_{y}L_{z}} - S_{IIV01} \frac{C_{V} - S_{VV02} - S_{IIV11}}{\Theta L_{x}L_{y}L_{z}} \Big) \Big) \\ & C_{I} = \frac{\alpha_{II}\alpha_{IV}}{\Theta L_{x}L_{y}L_{z}} S_{IIV00} + \frac{\alpha_{II}^{2}S_{II00}}{\Theta L_{x}L_{y}L_{z}} - \frac{S_{IIV0}}{\Theta L_{x}L_{y}L_{z}} - \frac{S_{IIV0}}{\Theta L_{x}L_{y}L_{z}} \Big) \Big) \\ \\ & C_{V} = \alpha_{II}\alpha_{IV}S_{IV00} + \alpha_{IV}^{2}S_{VV00} - S_{VV02} - S_{IIV11} \Big) \\ & E_{I} = \sqrt{8y + \Theta^{2}\frac{\alpha_{I}^{2}}{\alpha_{I}^{2}} - 4\Theta\frac{\alpha_{I}}{\alpha_{I}}} \Big) \Big) \\ \\ & C_{V} = \alpha_{II}\alpha_{IV}S_{IV00} + \alpha_{IV}^{2}S_{VV00} - S_{VV02} - S_{IIV11} \Big) \\ \\ & C_{I} = \frac{\alpha_{II}\alpha_{IV}}{\alpha_{I}^{2}} - \frac{\alpha_{II}^{2}}{\Theta L_{x}L_{y}L_{z}} - \frac{S_{IIV0}}{\Theta L_{x}L_{y}L_{z}} \Big) \\ \\ & C_{I} = \frac{\alpha_{II}\alpha_{IV}}{\alpha_{I}} \Big) \Big) \\ \\ & C_{I} = \frac{\alpha_{II}\alpha_{IV}}{\alpha_{I}} + \frac{\alpha_{IV$$

$$+\sqrt[3]{\sqrt{r^{2}+s^{3}}-r} - \sqrt[3]{\sqrt{r^{2}+s^{3}}+r}, r = \frac{\Theta^{3}b_{2}}{24b_{4}^{2}} \left(4b_{0} - \Theta L_{x}L_{y}L_{z}\frac{b_{1}b_{3}}{b_{4}}\right) - \frac{\Theta^{3}b_{2}^{3}}{54b_{4}^{3}} - b_{0}\frac{\Theta^{2}}{8b_{4}^{2}} \times \left(4\Theta b_{2} - \Theta^{2}\frac{b_{3}^{2}}{b_{4}}\right) - L_{x}^{2}L_{y}^{2}L_{z}^{2}\frac{\Theta^{4}b_{1}^{2}}{8b_{4}^{2}}, s = \Theta^{2}\frac{4b_{0}b_{4} - \Theta L_{x}L_{y}L_{z}b_{1}b_{3}}{12b_{4}^{2}} - \frac{\Theta b_{2}}{18b_{4}}$$

Further, we determine solutions of Eqs.(6), that is, components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections, we replace the required functions in the right sides of the equations by their not yet known average values  $\alpha_{i}$ . The substitution leads to the following result.

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2} = -K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}, \ \rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}$$

$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}, \ \rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2} = -K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

Integration of the left and the right sides of the above relations on time *t* leads to the following result.

$$u_{1x}(x,y,z,t) = u_{0x} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_{0}^{t}\int_{0}^{\theta} T(x,y,z,\tau) d\tau d\vartheta - -K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_{0}^{\infty}\int_{0}^{\theta} T(x,y,z,\tau) d\tau d\vartheta,$$

$$u_{1y}(x,y,z,t) = u_{0y} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_{0}^{t}\int_{0}^{\theta} T(x,y,z,\tau) d\tau d\vartheta - -K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_{0}^{\infty}\int_{0}^{\theta} T(x,y,z,\tau) d\tau d\vartheta,$$

$$u_{1z}(x,y,z,t) = u_{0z} + K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_{0}^{t}\int_{0}^{\theta} T(x,y,z,\tau) d\tau d\vartheta - -K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_{0}^{\infty} T(x,y,z,\tau) d\tau d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined using standard replacement of the required components on the following sums  $\alpha_i + u_i(x,y,z,t)^{[18-20]}$ . The replacement leads to the following result.

$$\rho\left(z\right)\frac{\partial^{2} u_{2x}\left(x, y, z, t\right)}{\partial t^{2}} = \left\{K\left(z\right) + \frac{5E\left(z\right)}{6\left[1 + \sigma\left(z\right)\right]}\right\}\frac{\partial^{2} u_{1x}\left(x, y, z, t\right)}{\partial x^{2}} + \left\{K\left(z\right) - \frac{E\left(z\right)}{3\left[1 + \sigma\left(z\right)\right]}\right\}$$
$$\times \frac{\partial^{2} u_{1y}\left(x, y, z, t\right)}{\partial x \partial y} + \frac{E\left(z\right)}{2\left[1 + \sigma\left(z\right)\right]}\left[\frac{\partial^{2} u_{1y}\left(x, y, z, t\right)}{\partial y^{2}} + \frac{\partial^{2} u_{1z}\left(x, y, z, t\right)}{\partial z^{2}}\right] - \frac{\partial T\left(x, y, z, t\right)}{\partial x} \times$$

$$\times K(z)\beta(z) + \left\{K(z) + \frac{E(z)}{3\left[1 + \sigma(z)\right]}\right\} \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial x \partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2} = \frac{E(z)}{2\left[1+\sigma(z)\right]} \left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x \partial y}\right] - \frac{\partial T(x,y,z,t)}{\partial y} \times$$

$$\times K(z)\beta(z) + \frac{\partial}{\partial z} \left\{ \frac{E(z)}{2\left[1 + \sigma(z)\right]} \left[ \frac{\partial u_{1y}(x, y, z, t)}{\partial z} + \frac{\partial u_{1z}(x, y, z, t)}{\partial y} \right] \right\} + \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y^2} \times$$

$$\times \left\{ \frac{5E(z)}{12\left[1+\sigma(z)\right]} + K(z) \right\} + \left\{ K(z) - \frac{E(z)}{6\left[1+\sigma(z)\right]} \right\} \frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y \partial z} + K(z) \frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x \partial y}$$

$$\rho(z)\frac{\partial^{2}u_{2z}(x,y,z,t)}{\partial t^{2}} = \frac{E(z)}{2\left[1+\sigma(z)\right]} \left[\frac{\partial^{2}u_{1z}(x,y,z,t)}{\partial x^{2}} + \frac{\partial^{2}u_{1z}(x,y,z,t)}{\partial y^{2}} + \frac{\partial^{2}u_{1x}(x,y,z,t)}{\partial x \partial z} + \frac{\partial^{2}u_{1x}(x,y,z,t)}{\partial x \partial z}\right]$$

$$+\frac{\partial^{2}u_{1y}(x,y,z,t)}{\partial y \partial z} + \frac{\partial}{\partial z} \left\{ K(z) \left[ \frac{\partial u_{1x}(x,y,z,t)}{\partial x} + \frac{\partial u_{1y}(x,y,z,t)}{\partial y} + \frac{\partial u_{1x}(x,y,z,t)}{\partial z} \right] \right\} + \frac{E(z)}{6[1+\sigma(z)]} \frac{\partial}{\partial z} \left[ 6 \frac{\partial u_{1z}(x,y,z,t)}{\partial z} - \frac{\partial u_{1x}(x,y,z,t)}{\partial x} - \frac{\partial u_{1y}(x,y,z,t)}{\partial y} - \frac{\partial u_{1z}(x,y,z,t)}{\partial z} \right] - \frac{\partial u_{1x}(x,y,z,t)}{\partial x} - \frac{\partial u_{1y}(x,y,z,t)}{\partial y} - \frac{\partial u_{1z}(x,y,z,t)}{\partial z} \right]$$

Integration of the left and right sides of the above relations on time t leads to the following result

$$u_{2x}(x, y, z, t) = \frac{1}{\rho(z)} \left\{ K(z) + \frac{5E(z)}{6[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial x^2} \int_0^t \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \left\{ K(z) - \frac{1}{\rho(z)} \right\} \frac{\partial^2}{\partial x^2} \int_0^t \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \left\{ K(z) - \frac{1}{\rho(z)} \right\} \frac{\partial^2}{\partial x^2} \int_0^t \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta$$

$$\begin{split} &-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\frac{\partial^2}{\partial x\partial y}\int_0^z h_{u_y}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta + \frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^z h_{u_y}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta + \\ &+\frac{\partial^2}{\partial z^2}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \right]\frac{1}{1+\sigma(z)} + \frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \left\{K(z) + \\ &+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right] - K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^z h_0^{u_y}(T(x,y,z,\tau)\,d\tau\,d\theta - \frac{\partial^2}{\partial x^2}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \times \\ &\times\frac{1}{\rho(z)}\left\{K(z) + \frac{5E(z)}{6\left[1+\sigma(z)\right]}\right] - \left\{K(z) - \frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\frac{\partial^2}{\partial x\partial z}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \times \\ &\times\frac{1}{\rho(z)}\left\{K(z) + \frac{5E(z)}{6\left[1+\sigma(z)\right]}\right] - \frac{\partial}{\partial y^2}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta + \frac{\partial^2}{\partial z^2}\int_0^z h_{u_z}^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \times \\ &\times\frac{1}{\rho(z)}\left\{K(z) + \frac{5E(z)}{3\left[1+\sigma(z)\right]}\right]\frac{\partial^2}{\partial y^2}\int_0^{u_y} h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + \frac{\partial^2}{\partial z^2}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta \\ &\times\frac{1}{\rho(z)}\left\{K(z) + \frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\frac{\partial^2}{\partial x\partial z}\int_0^{u_y} h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + h_{u_z}+K(z)\frac{\beta(z)}{\rho(z)}\times \\ &\times\frac{\partial}{\partial x}\int_0^z h_0^{u_y}(x,y,z,\tau)\,d\tau\,d\theta \\ &u_{1y}(x,y,z,\tau)\,d\tau\,d\theta \\ &\times\frac{1}{1+\sigma(z)} + \frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^z h_{u_y}(x,y,z,\tau)\,d\tau\,d\theta + \frac{1}{\rho(z)}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]} + K(z)\right\}\times \\ &\times\frac{\partial^2}{\partial y^2}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + \frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\right\right]\frac{\partial}{\partial z}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + \frac{\partial}{\partial z}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + \\ &+\frac{\partial}{\partial y}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta + \frac{1}{2\rho(z)}\frac{\partial}{\partial z}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \left\{\frac{E(z)}{6\left[1+\sigma(z)\right]} - K(z)\frac{\partial}{\rho(z)}\frac{\partial}{\partial z}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \left\{\frac{E(z)}{6\left[1+\sigma(z)\right]} - K(z)\frac{\partial}{\rho(z)}\frac{\partial}{\partial z}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{E(z)}{6\left[1+\sigma(z)\right]} - \\ &-K(z)\left\{\frac{\partial}{\partial x\partial y}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{E(z)}{\rho(z)}\right\} + \\ &\frac{\partial^2}{\partial x\partial y}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{E(z)}{6\left[1+\sigma(z)\right]} + \\ &\frac{\partial}{\partial x\partial y}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{E(z)}{\rho(z)}\right\} + \\ &\frac{\partial^2}{\partial x\partial y}\int_0^z h_{u_z}(x,y,z,\tau)\,d\tau\,d\theta - \frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^z h_{u_z}(x$$

$$+ K(z) \bigg\} - \frac{\partial}{\partial z} \bigg\{ \frac{E(z)}{1+\sigma(z)} \bigg[ \frac{\partial}{\partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_{0}^{\infty} \int_{0}^{0} u_{1z}(x, y, z, \tau) d\tau d\vartheta \bigg] \bigg\} \times \\ \times \frac{1}{2\rho(z)} - \frac{1}{\rho(z)} \bigg\{ K(z) - \frac{E(z)}{6[1+\sigma(z)]} \bigg\} \frac{\partial^2}{\partial y \partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1y}(x, y, z, \tau) d\tau d\vartheta + u_{0y} \\ u_z(x, y, z, t) = \frac{E(z)}{2[1+\sigma(z)]} \bigg[ \frac{\partial^2}{\partial x^2} \int_{0}^{\infty} \int_{0}^{0} u_{1z}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y^2} \int_{0}^{\infty} \int_{0}^{0} u_{1z}(x, y, z, \tau) d\tau d\vartheta + \\ + \frac{\partial^2}{\partial x \partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y \partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1y}(x, y, z, \tau) d\tau d\vartheta \bigg] \frac{1}{\rho(z)} + \frac{1}{\rho(z)} \times \\ \times \frac{\partial}{\partial z} \bigg\{ K(z) \bigg[ \frac{\partial}{\partial x} \int_{0}^{\infty} \int_{0}^{0} u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_{0}^{\infty} \int_{0}^{0} u_{1x}(x, y, z, \tau) d\tau d\vartheta + \\ + \frac{\partial}{\partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1x}(x, y, z, \tau) d\tau d\vartheta \bigg] \bigg\} + \frac{1}{6\rho(z)} \frac{\partial}{\partial z} \bigg\{ \frac{E(z)}{1+\sigma(z)} \bigg[ 6 \frac{\partial}{\partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1z}(x, y, z, \tau) d\tau d\vartheta - \\ - \frac{\partial}{\partial x} \int_{0}^{\infty} \int_{0}^{0} u_{1x}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial y} \int_{0}^{\infty} \int_{0}^{0} u_{1y}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial z} \int_{0}^{\infty} \int_{0}^{0} u_{1y}(x, y, z, \tau) d\tau d\vartheta - \\ - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_{0}^{\infty} \int_{0}^{0} T(x, y, z, \tau) d\tau d\vartheta + u_{0z} \bigg\}$$

In the framework of this paper, we determine the concentration of radiation defects and components of displacement vector using the second-order approximation framework method of averaging of function corrections. The approximation is usually enough good approximation to make a qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.



**Figure 2:** Typical distributions of concentration of interstitial atoms (distributions of concen-trations of other types of defects are similar to distributions of concentration of interstitial atoms). Increasing the number of curves corresponds to increasing the annealing time

### DISCUSSION

In this section, we will analyze the redistribution of radiation defects during annealing of them with into account relaxation of mismatch-induced stresses. Typical distributions of concentrations of point radiation defects (for both types of radiation defects, the distribution of concentrations will be similar to each other) in the considered multilayer structures are shown on Figure 2 for the case, when the diffusion coefficient of defects in the buffer layer is larger than the same coefficient in the epitaxial layer. One can find qualitatively similar distributions of concentration for point defect complexes. It follows from this figure, that the inhomogeneity of properties of the multilayer structure could decrease the quantity of radiation defects in the working region of integrated circuits of the multilayer structure.

### CONCLUSION

In this paper, we introduce an approach for the organization of a drain of radiation defects, which were generated during ion doping or other types of radiation processing of the working area of integrated circuits, manufactured framework of a multilayer structure. The approach is based on the difference of properties of materials of the layers of the considered multilayer structure. An analytical approach for analysis of mass and heat transfer in multilayer structures was introduced with account of the spatial and temporal variations of their parameters, as well as the non-linearity of the processes under consideration.

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